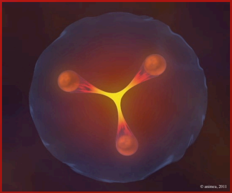
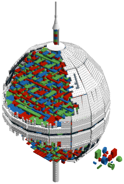
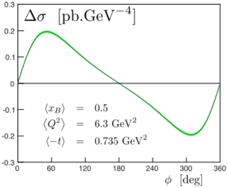
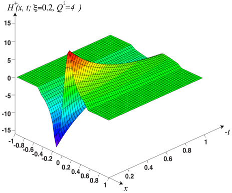
DE LA RECHERCHE À L'INDUSTRIE



Proton tomography:
Towards new information on the strong interaction

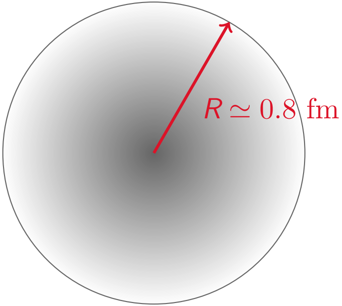




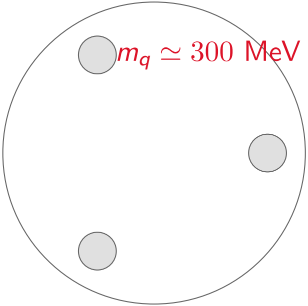


DROITE workshop on tomography | Hervé MOUTARDE

Jan. 27th, 2017



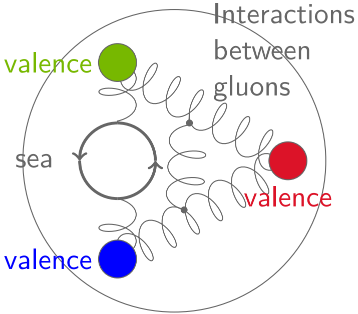
- **Composite** object with an **electric charge** spread over a spherical region.



- **Composite** object with an **electric charge** spread over a spherical region.
- Quark model description: **nonrelativistic** bound state of **3 massive quarks**.

Hadrons and partons.

The nucleon: a **quantum relativistic** system of **confined** particles.



- **Composite** object with an **electric charge** spread over a spherical region.
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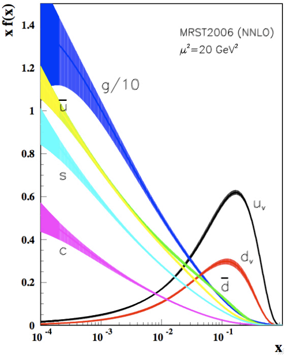

Partons number densities vs longitudinal momentum

- **Composite** object with an **electric charge** spread over a spherical region.
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- **Arbitrarily many** quarks, antiquarks and gluons in nucleons.

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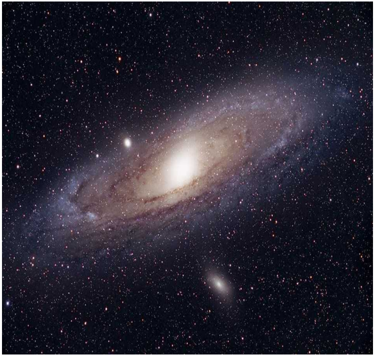
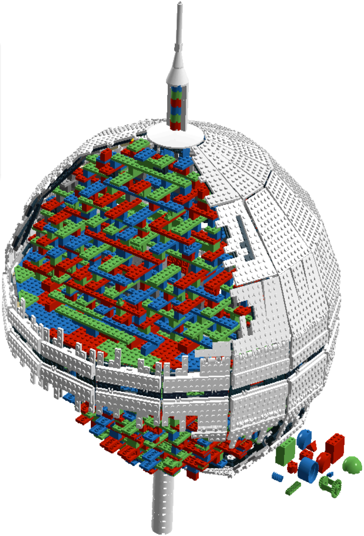


QCD generates $\gtrsim$ 90 % of the visible universe mass

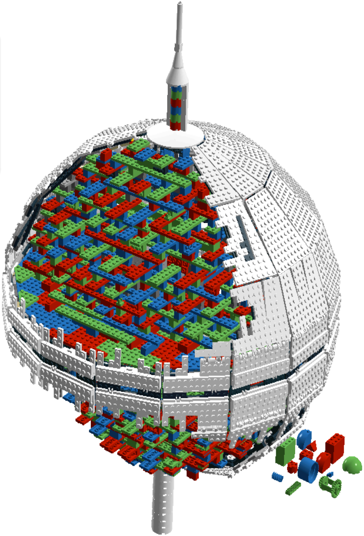
- **Composite** object with an **electric charge** spread over a spherical region.
- Quark model description: **nonrelativistic** bound state of **3 massive quarks**.
- Modern description (QCD): **relativistic** bound state of **colored light** quarks and **massless** gluons (**partons**).
- **Arbitrarily many** quarks, antiquarks and gluons in nucleons.
- QCD: few **principles**, wide **scope** and puzzling **properties**:
  - ✓ Asymptotic freedom,
  - X Confinement.

How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?



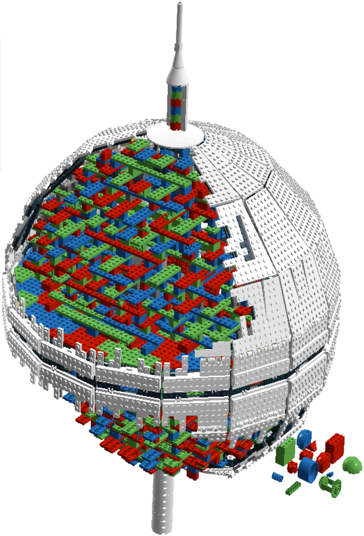
How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

Mass?



How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

Mass?
Spin?

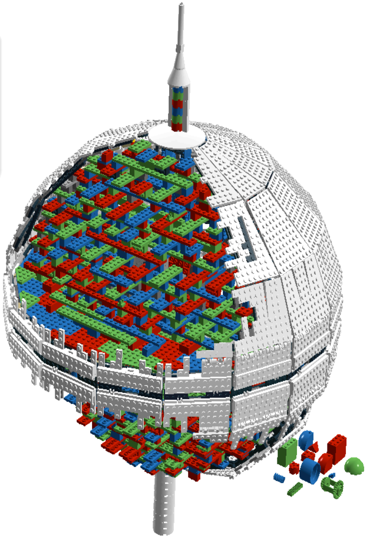


Key questions.

Embracing QCD dynamics from the nucleon structure viewpoint.

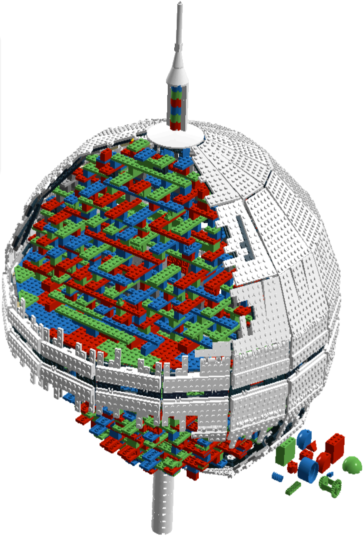
How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

Mass?
Spin?
Charge?



How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

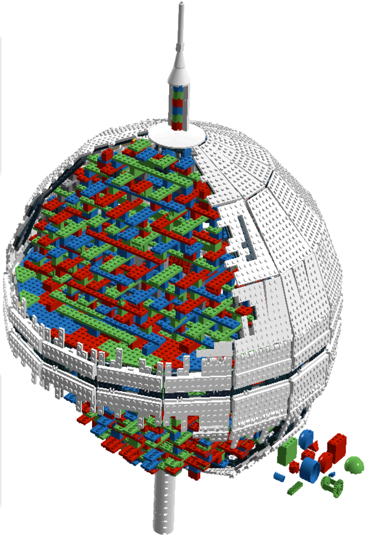
Mass?
Spin?
Charge?
...



How can we recover the well-known characterics of the nucleon from the properties of its **colored building blocks**?

Mass?
Spin?
Charge?
...

What are the relevant **effective degrees of freedom** and **effective interaction** at large distance?



1 Definitions and properties

Theoretical constraints on Generalized Parton Distributions.

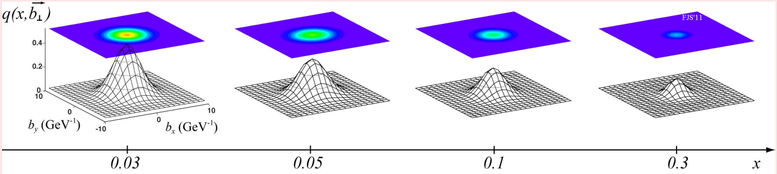
2 Modeling

Generalized Parton Distribution modeling and the inverse Radon transform

3 Experimental data analysis

Experimental data analysis: deconvolution problem

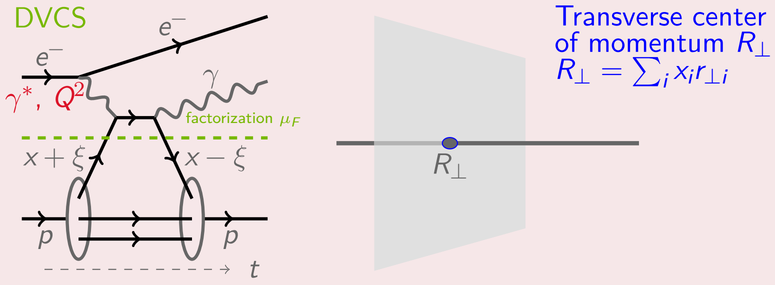
How can we make this picture? What do we learn from it?



Definitions and properties

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

Deeply Virtual Compton Scattering (DVCS)



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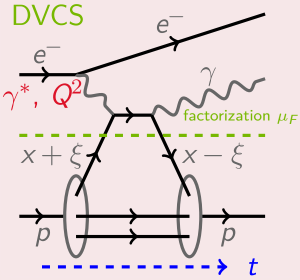

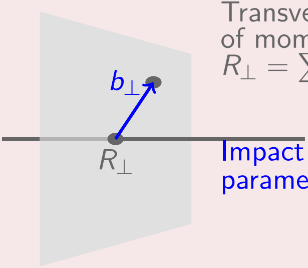

Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

Impact parameter $b_\perp$

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
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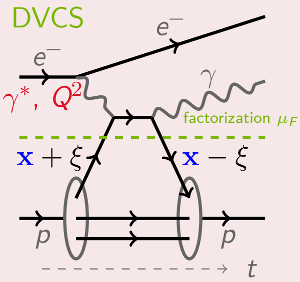
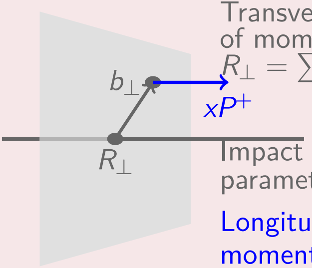
Deeply Virtual Compton Scattering (DVCS)





Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

Impact parameter $b_\perp$

Longitudinal momentum xP^+

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
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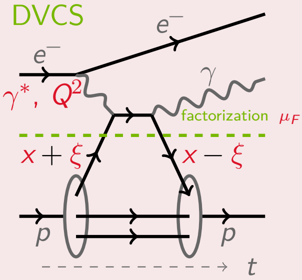
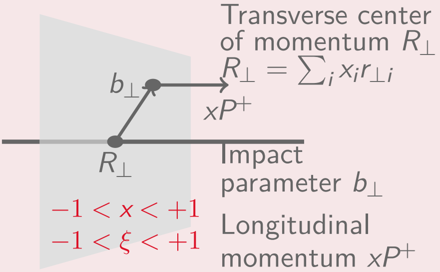
Deeply Virtual Compton Scattering (DVCS)





- **24 GPDs** $F^i(x, \xi, t, \mu_F)$ for each parton type $i = g, u, d, \ldots$ for leading and sub-leading twists.

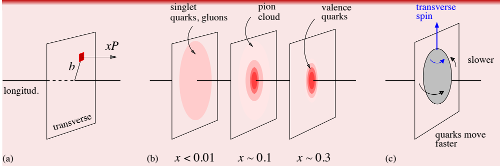
- **Probabilistic interpretation** of Fourier transform of $\mathrm{GPD}(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2}\Big[H(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M}\frac{\partial E}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda\lambda_N \tilde{H}(x, 0, b_\perp^2)\Big]$$

- Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin $S_\perp$.

Burkardt, Phys. Rev. **D62**, 071503 (2000)

Can we obtain this picture from exclusive measurements?



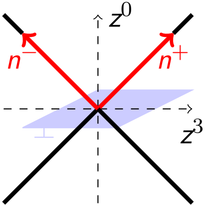
Weiss, AIP Conf. Proc. **1149**, 150 (2009)

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**

$$H^q(x, 0, 0) = q(x)$$

$$H^q_\pi(x,\xi,t) \quad = \quad \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

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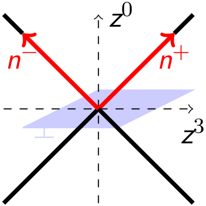

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- PDF **forward limit**
- Form factor **sum rule**

$$\int_{-1}^{+1} dx\, H^q(x,\xi,t) = F_1^q(t)$$

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

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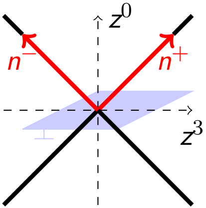

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- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.

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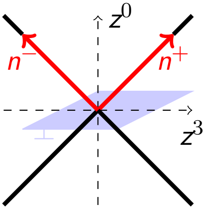

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Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

Polynomiality

$$\int_{-1}^{+1} dx\, x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

Polynomiality

Lorentz covariance

- **Polynomiality**

Lorentz covariance

- **Positivity**

$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

- **Polynomiality**

Lorentz covariance

- **Positivity**

Positivity of Hilbert space norm

- **Polynomiality**

  Lorentz covariance

- **Positivity**

  Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

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  Relativistic quantum mechanics

- **Polynomiality**

Lorentz covariance

- **Positivity**

Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

$$H^q(x, \xi = 1, t = 0) = \frac{1}{2}\phi_\pi^q\left(\frac{1+x}{2}\right)$$

- **Polynomiality**

  Lorentz covariance

- **Positivity**

  Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

  Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

  Dynamical chiral symmetry breaking

- **Polynomiality**

  Lorentz covariance

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  Positivity of Hilbert space norm

- H^q has support $x \in [-1, +1]$.

  Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

  Dynamical chiral symmetry breaking

How can we implement *a priori* these theoretical constraints?

- There is no known GPD parameterization **relying only on first principles**.
- In the following, focus on **polynomiality** and **positivity**.

Generalized Parton Distribution modeling and the inverse Radon transform

- Representation of GPD:

$$H^q(x,\xi,t) = \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t)+\xi\, G^q(\beta,\alpha,t)\big)$$

- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.
- **Gauge**: any representation (F^q, G^q) can be recast in one representation with a single DD f^q:

$$H^q(x,\xi,t) = x \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, f^q_{\mathrm{BMKS}}(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

Belitsky *et al.*, Phys. Rev. **D64**, 116002 (2001)

$$H^q(x,\xi,t) = (1-x) \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, f^q_{\mathrm{P}}(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

Pobylitsa, Phys. Rev. **D67**, 034009 (2003)
Müller, Few Body Syst. **55**, 317 (2014)

- Choose $F^q(\beta, \alpha) = 3\beta\theta(\beta)$ ad $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$:

$$H^q(x, \xi) = 3x \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi)$$

- Simple analytic expressions for the GPD:

$$\begin{aligned} H(x, \xi) &= \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1, \\ H(x, \xi) &= \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1. \end{aligned}$$

- Compute first Mellin moments.

n	$\int_{-\xi}^{+\xi} \mathrm{d}x\, x^n H(x,\xi)$	$\int_{+\xi}^{+1} \mathrm{d}x\, x^n H(x,\xi)$	$\int_{-\xi}^{+1} \mathrm{d}x\, x^n H(x,\xi)$
0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1
1	$\frac{1+\xi+\xi^2-3\xi^3}{2(1+\xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$
2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$
3	$\frac{1+\xi+\xi^2+\xi^3+\xi^4-5\xi^5}{5(1+\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$
4	$\frac{1+\xi+\xi^2+\xi^3+\xi^4+\xi^5-6\xi^6}{7(1+\xi)}$	$\frac{6\xi^6}{7(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{7}$

- Expressions get more complicated as n increases... But they always yield polynomials!

- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_\perp]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \ldots, x_N, \mathbf{k}_{\perp N}) \, |\beta, k_1, \ldots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x} \mathrm{d}\bar{\mathbf{k}}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) \big(\psi_N^{(\beta,\lambda)}\big)^* (\hat{x}', \hat{\mathbf{k}}'_\perp) \psi_N^{(\beta,\lambda)}(\tilde{x}, \tilde{\mathbf{k}}_\perp)$$

with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

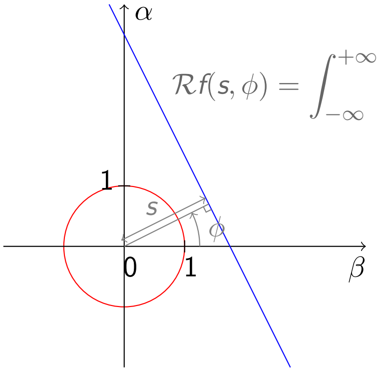
- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N- and $(N+2)$-body LFWFs.

- Physical picture.
- Positivity relations are fulfilled **by construction**.
- Implementation of **symmetries of N-body problems**.

What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at** $x = \pm\xi$ and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate the above requirements**.

Diehl, Phys. Rept. **388**, 41 (2003)



For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s,\phi) = \int_{-\infty}^{+\infty} \mathrm{d}\beta \mathrm{d}\alpha\, f(\beta,\alpha)\delta(s - \beta\cos\phi - \alpha\sin\phi)$$

and:

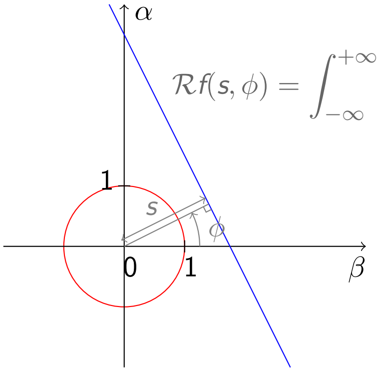
$$\mathcal{R}f(-s,\phi) = \mathcal{R}f(s,\phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Belistky *et al.* gauge

$$\frac{\sqrt{1+\xi^2}}{x} H(x,\xi) = \mathcal{R}f_{\mathrm{BMKS}}(s,\phi) ,$$



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Relation to GPDs:

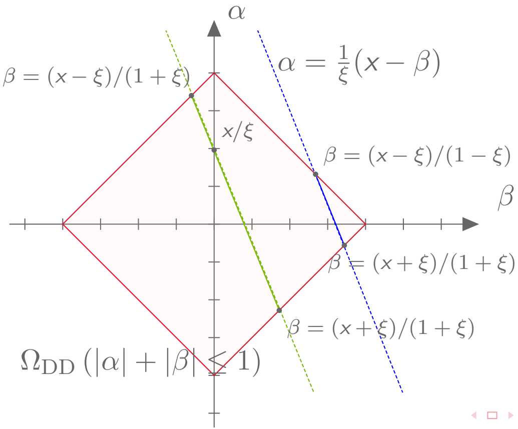
$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x} H(x,\xi) = \mathcal{R}f_{\mathrm{P}}(s,\phi) \,,$$

DGLAP and ERBL regions

$$(x, \xi) \in \text{DGLAP} \Leftrightarrow |s| \geq |\sin\phi| ,$$
$$(x, \xi) \in \text{ERBL} \Leftrightarrow |s| \leq |\sin\phi| .$$



- Each point (β, α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.
- Expressed in **support theorem**.

Theorem

Let f be a compactly-supported locally summable function defined on $\mathbb{R}^2$ and $\mathcal{R}f$ its Radon transform.
Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 such that:

$$\textit{for all } s > s_0 \textit{ and } \omega \in U_0 \quad \mathcal{R}f(s, \omega) = 0 \ .$$

Then $f(\aleph) = 0$ on the half-plane $\langle \aleph | \omega_0 \rangle > s_0$ of $\mathbb{R}^2$.

Consider a GPD H being zero on the DGLAP region.

- Take ϕ_0 and s_0 *s.t.* $\cos\phi_0 \neq 0$ and $|s_0| > |\sin\phi_0|$.
- Neighborhood U_0 of ϕ_0 *s.t.* $\forall \phi \in U_0 \quad |\sin \ \phi| < |s_0|$.
- The underlying DD f has a zero Radon transform for all $\phi \in U_0$ and $s > s_0$ (DGLAP).
- Then $f(\beta, \alpha) = 0$ for all $(\beta, \alpha) \in \Omega_{\mathrm{DD}}$ with $\beta \neq 0$.
- Extension **unique** up to adding a **D-term**: $\delta(\beta)D(\alpha)$.

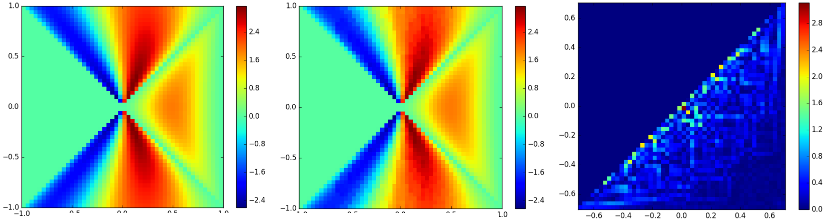
Fully discrete case

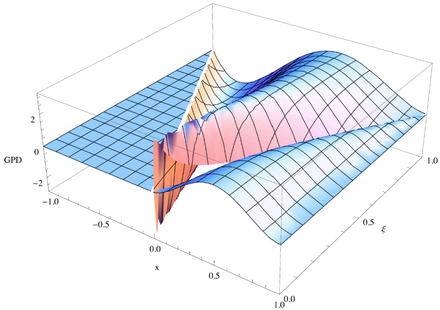
Assume f piecewise-constant with values f_m for $1 \leq m \leq M$.
For a collection of lines $(L_n)_{1 \leq n \leq N}$ crossing Ω_{DD}, the Radon transform writes:

$$g_n = \mathcal{R}f = \int_{L_n} f = \sum_{m=1}^{M} f_m \times \mathrm{Measure}(L_n \cap C_m) \quad \text{for } 1 \leq n \leq N$$

And if the input data are inconsistent?

- Instead of solving $g = \mathcal{R}f$, find f such that $\|g - \mathcal{R}f\|_2$ is **minimum**.
- The solution **always exists**.
- The input data are **inconsistent** if $\|g - \mathcal{R}f\|_2 > 0$.

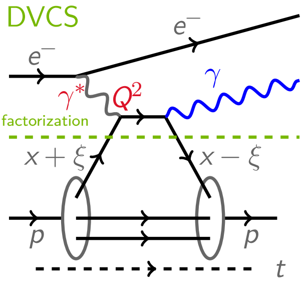




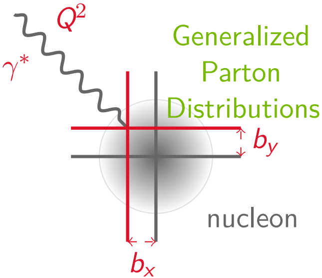
Chouika

Work in progress

Experimental data analysis: deconvolution problem



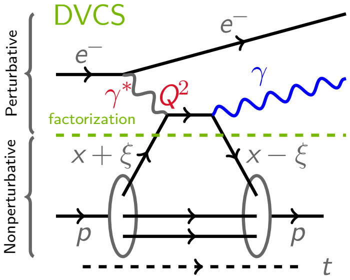
DVCS
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t

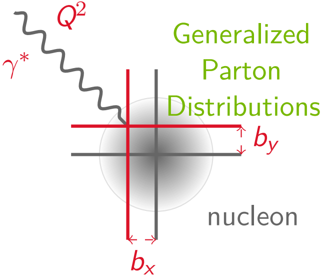


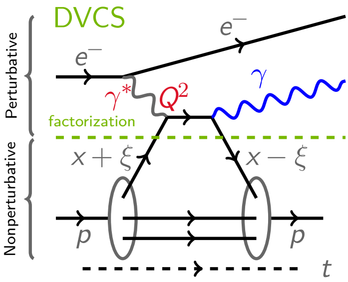
Q^2
γ^*
Generalized
Parton
Distributions
b_y
nucleon
b_x

Exclusive processes of current interest (1/2).

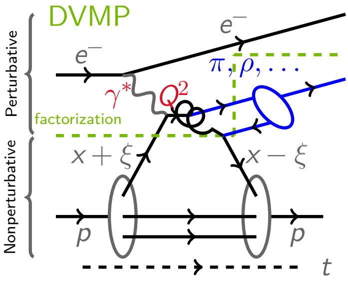
Factorization and universality.



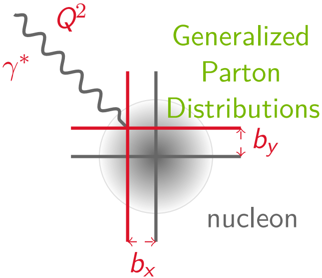




DVCS
Perturbative
Nonperturbative
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t



DVMP
Perturbative
Nonperturbative
e^-
e^-
$\pi, \rho, \ldots$
γ^*
Q^2
factorization
$x+\xi$
$x-\xi$
p
p
t



Q^2
γ^*
Generalized Parton Distributions
b_y
nucleon
b_x

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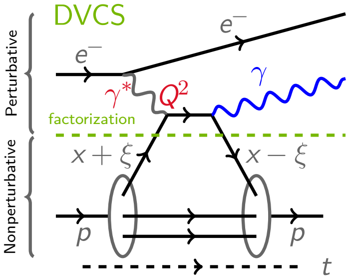

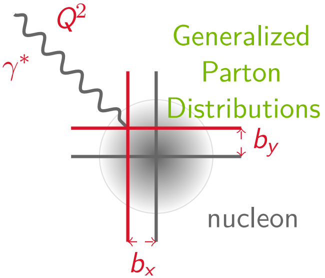

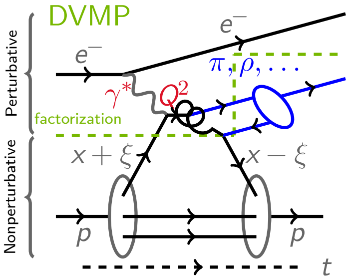

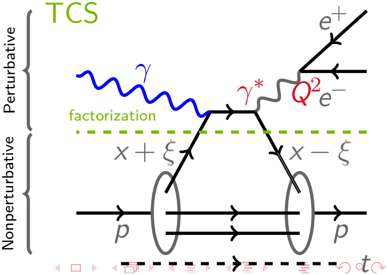

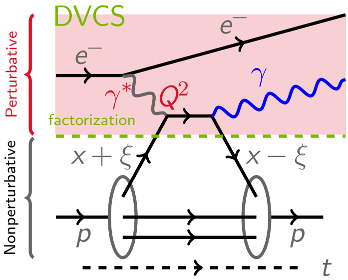
DVCS
Perturbative
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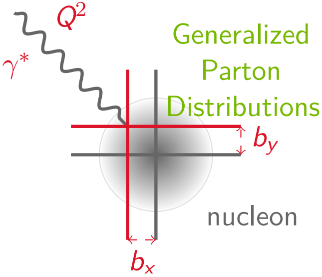
p
p
t


γ^*
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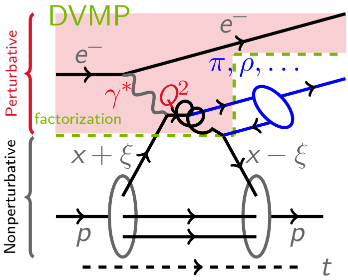
DVMP
Perturbative
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p
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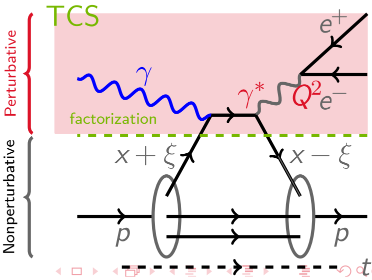
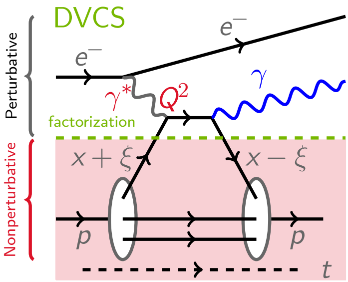
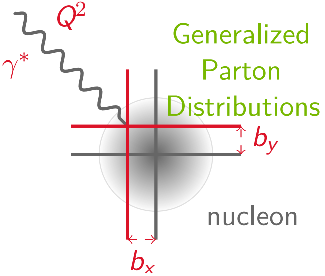
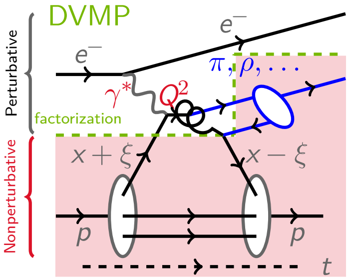

TCS
Perturbative
Nonperturbative
e^+
γ
γ^*
Q^2
e^-
factorization
$x+\xi$
$x-\xi$
p
p
t

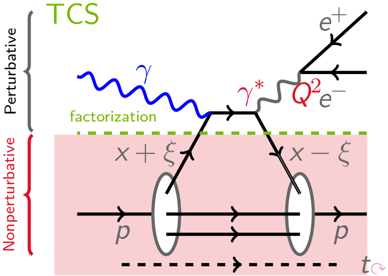
Exclusive processes of current interest (1/2).

Factorization and universality.









Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

- CFF $\mathcal{F}$ is a **complex function**.

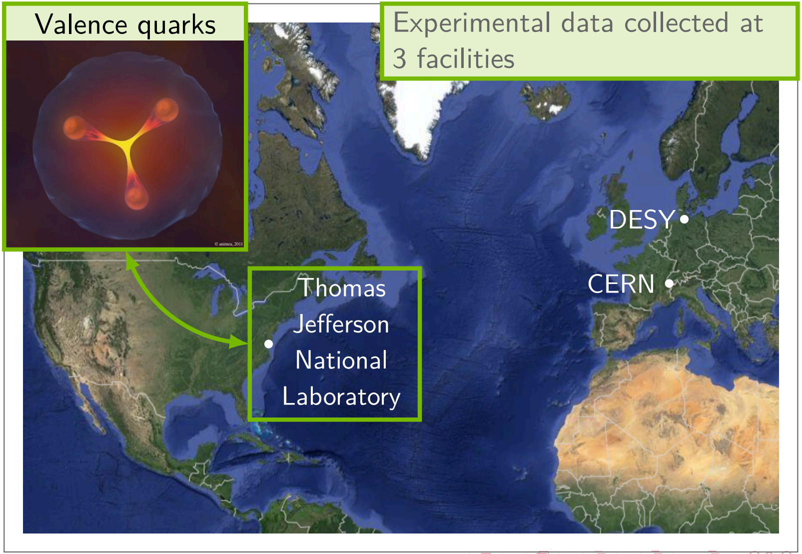
Need for global fits of world data.

Different facilities will probe different kinematic domains.

Experimental data collected at 3 facilities

Need for global fits of world data.

Different facilities will probe different kinematic domains.



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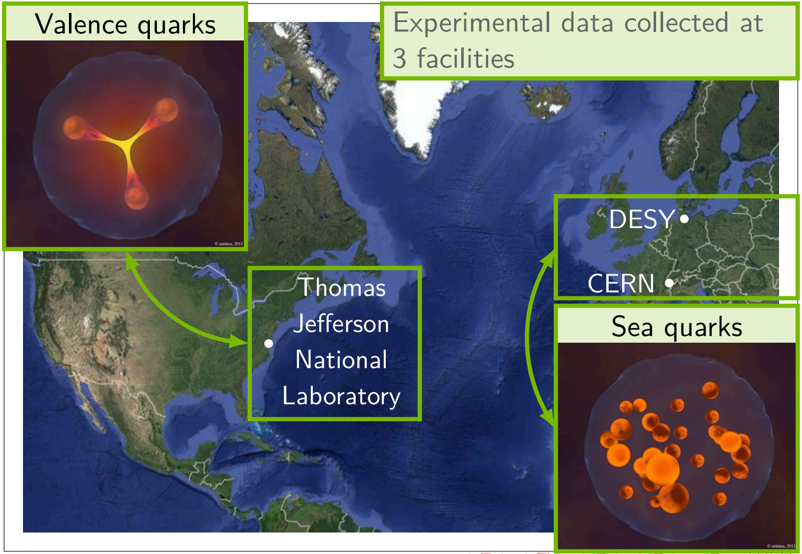
Different facilities will probe different kinematic domains.



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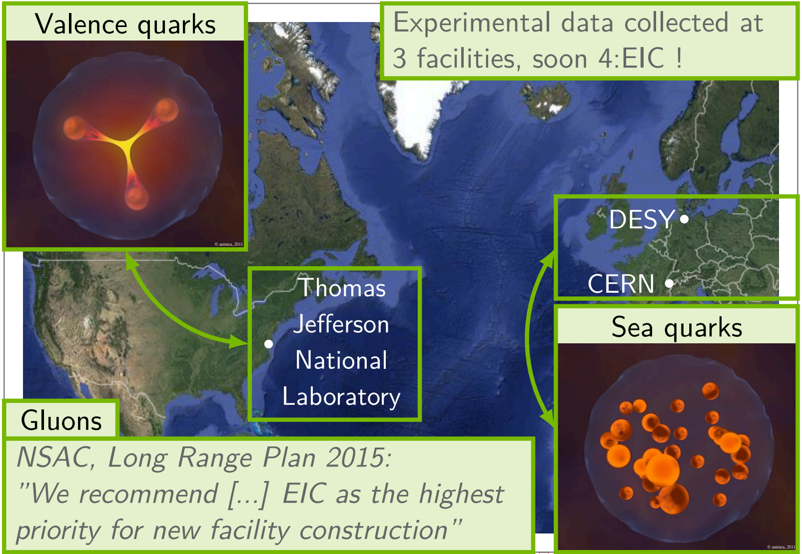
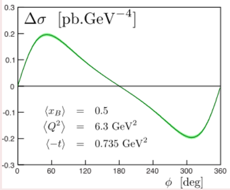


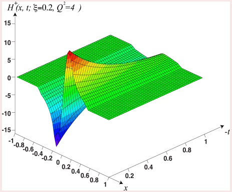
Imaging the nucleon. How?

Extracting GPDs is not enough...Need to extrapolate!

1. Experimental data fits



2. GPD extraction



3. Nucleon imaging

Images from Guidal et al.,
Rept. Prog. Phys. 76 (2013) 066202

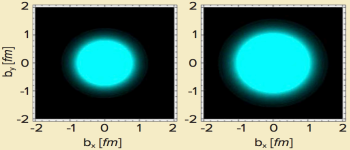
Reaching for the Horizon

The 2015 Long Range Plan for Nuclear Science

Sidebar 2.2: The First 3D Pictures of the Nucleon

A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Now physicists are using the principles behind the procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new concept in nuclear physics called generalized parton distributions.

An intense beam of high-energy electrons can be used

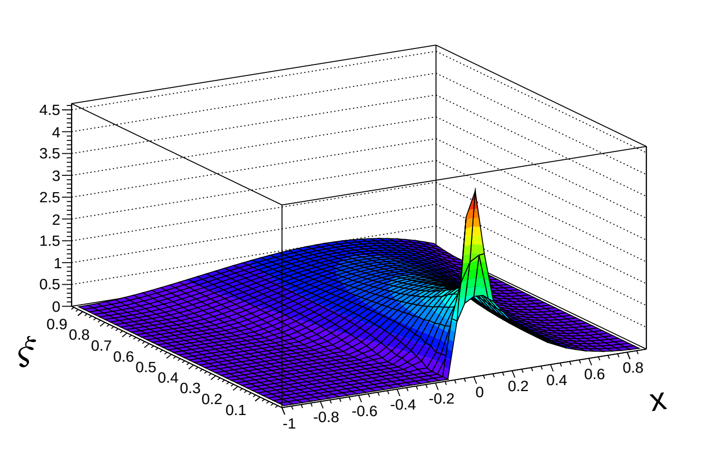


1. **Extract** $H(x, \xi, t, \mu_F^{\text{ref}})$ from experimental data.
2. **Extrapolate** to vanishing skewness $H(x, 0, t, \mu_F^{\text{ref}})$.
3. **Extrapolate** $H(x, 0, t, \mu_F^{\text{ref}})$ up to infinite t.
4. **Compute** 2D Fourier transform in transverse plane:

$$H(x, b_\perp) = \int_0^{+\infty} \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0(|b_\perp||\Delta_\perp|) H(x, 0, -\Delta_\perp^2)$$

5. **Propagate** uncertainties.
6. **Control** extrapolations with an accuracy matching that of experimental data with **sound** GPD models.

GPD H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2.

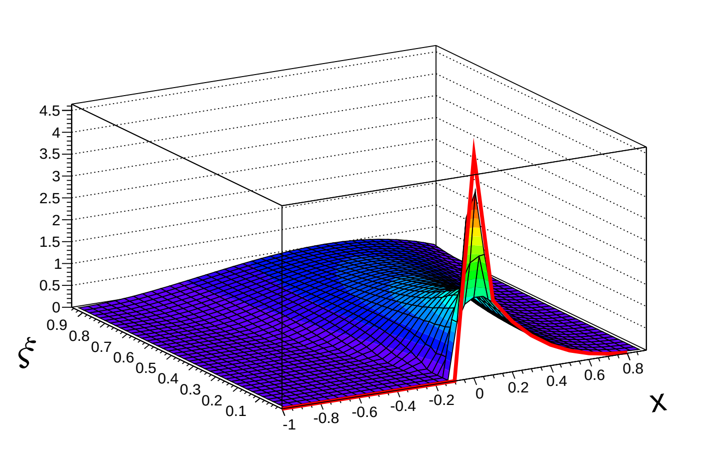


GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted kinematic domain.

Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.

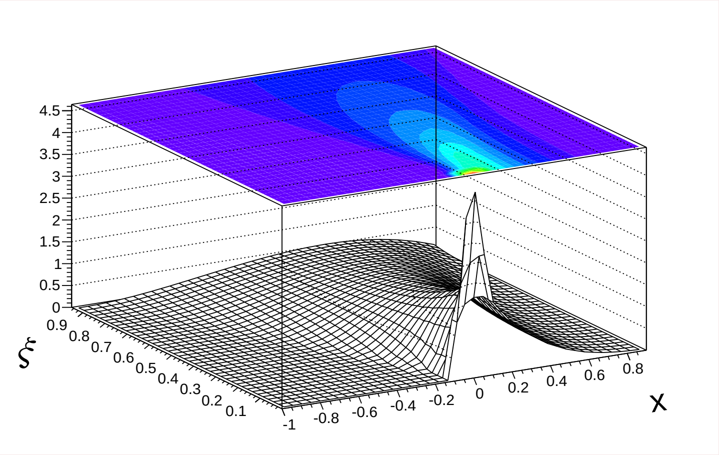


GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

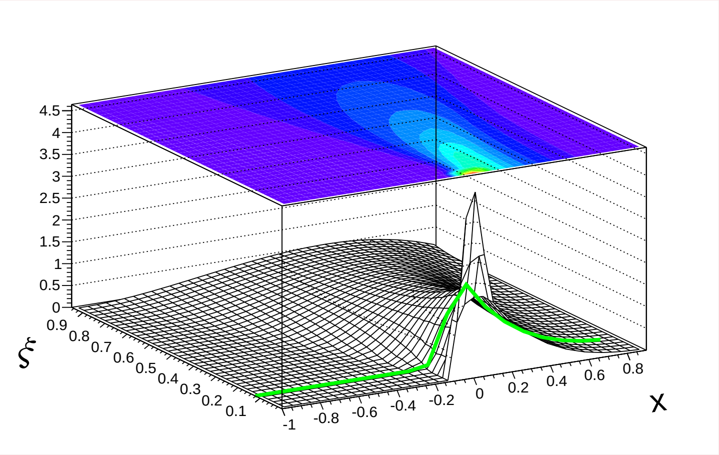
Direct experimental access to a restricted kinematic domain.

What is the physical region?



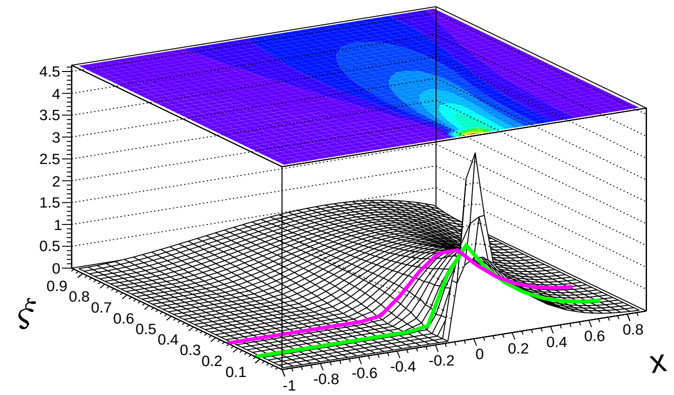
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

$\xi_{\min}$ from finite beam energy.



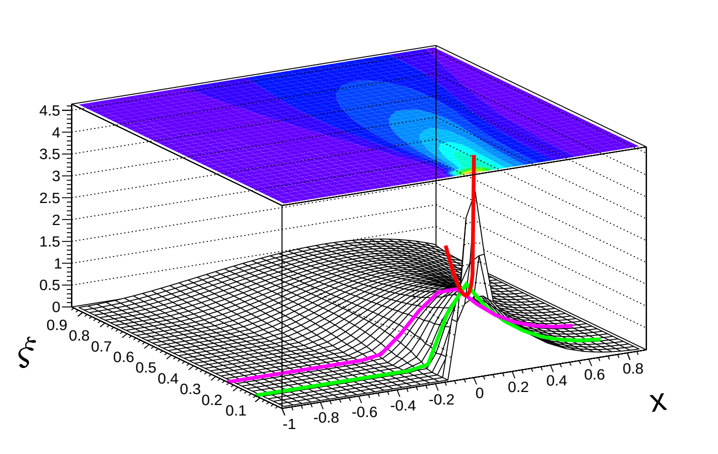
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

$\xi_{\max}$ from kinematic constraint on 4-momentum transfer.



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

The cross-over line $x = \xi$.



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

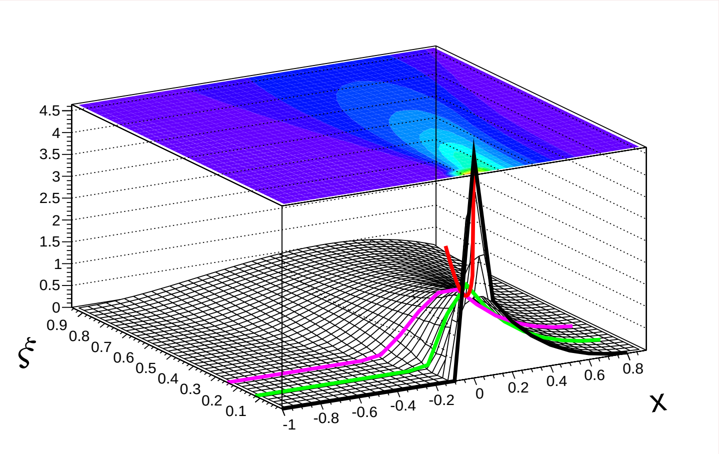
From principles to actual data.

Direct experimental access to a restricted kinematic domain.

CEA - Saclay

Proton tomography

The black curve is what is needed for transverse plane imaging!

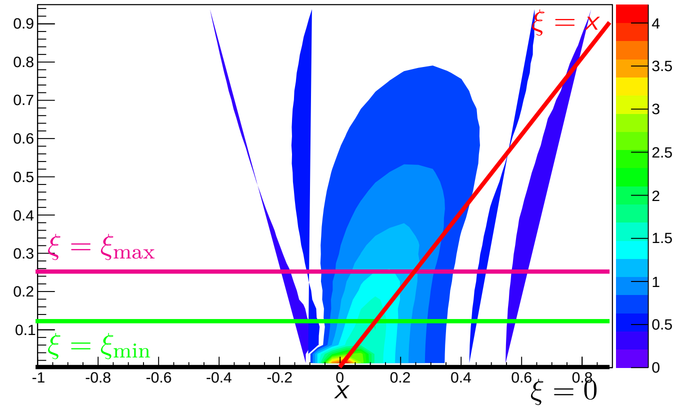


GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted kinematic domain.

Density plot of H at $t = -0.23\ \mathrm{GeV}^2$ and $Q^2 = 2.3\ \mathrm{GeV}^2$

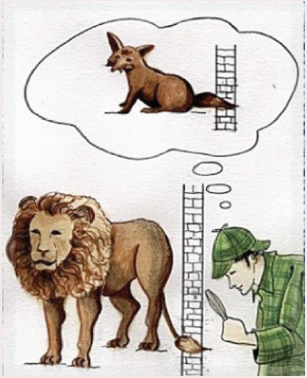


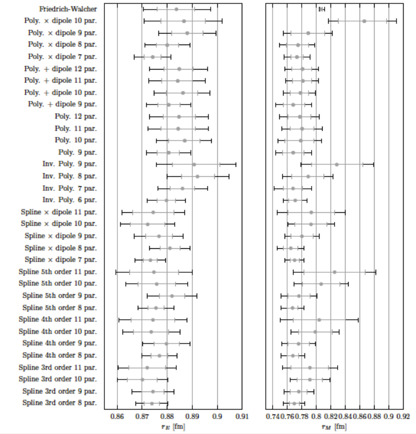
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Support theorem.

We don't need to know the GPD everywhere to image the proton!

Extrapolations...





Bernauer *et al.*(A1 Coll.), Phys. Rev. **C90**, 015206 (2014)

Support theorem.

We don't need to know the GPD everywhere to image the proton!

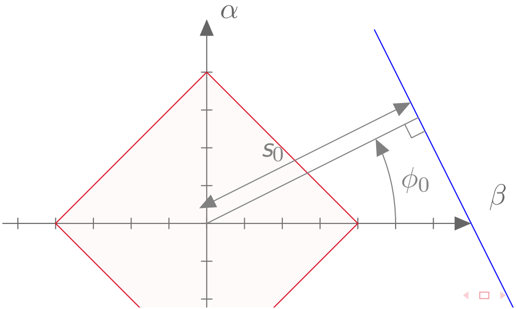
Theorem

Let f be a compactly-supported summable function defined on $\mathbb{R}^2$ and $\mathcal{R}f$ its Radon transform.

Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 s.t.:

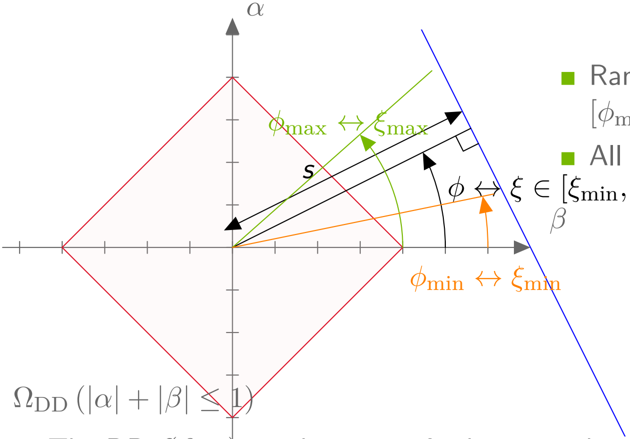
$$\textit{for all } s > s_0 \textit{ and } \omega \in U_0 \quad \mathcal{R}f(s, \omega) = 0 \, .$$

Then $f(\aleph) = 0$ on the half-plane $\langle \aleph | \omega_0 \rangle > s_0$ of $\mathbb{R}^2$.



- $\omega_0 = (\cos \phi_0, \sin \phi_0)$.
- $\aleph = (\beta, \alpha)$.

- Assume deconvolution of CFF achieved.
- Data: $H(x, \xi)$ for all $x \in [-1, +1]$ and $\xi \in [\xi_{\min}, \xi_{\max}]$.



- Range $\phi \in [\phi_{\min}, \phi_{\max}]$.
- All values of s.

- The DD $f(\beta, \alpha)$ can be **uniquely** determined.
- $H(x, \xi = 0)$ **uniquely** constrained by **Lorentz covariance**!

Conclusion

- **Good theoretical control** on the path between GPD models and experimental data.
- Success of physics program requires new GPD models with **proper implementations of symmetries**.
- Inverse Radon transform is essential to build models fulfilling *a priori* all theoretical constraints **in a generic way**.
- The solution of the **deconvolution problem** may follow.

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